BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

B69 11048

SUBJECT: Concept for a Two-Dimensional Analog Integrator, with Application to Slope-to-Height Conversion - Case 340

DATE: November 18, 1969

FROM: H. W. Radin

ABSTRACT

A concept is suggested for a two-dimensional electro-optical integrator, which could serve as a building block for a two-dimensional analog computer. One application of a simple version, and in fact the original motivation for the suggestion, is to the problem of determining lunar surface elevation from surface slope data (using the photometric function). The development of the idea is couched in these terms, and a possible way of dealing with the nonlinearity of the photometric function is suggested.

(NASA-CR-109063) CONCEPT FOR A TWO-DIMENSIONAL ANALOG INTEGRATOR, WITH APPLICATION TO SLOPE-TO-HEIGHT CONVERSION (Bellcomm, Inc.) 15 p

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MEMORANDUM FOR FILE

1.0 INTRODUCTION

The principal rival to stereo photography for lunar slope and elevation mapping has been photometry, utilizing the relationship between surface brightness and local slope. The principal objection, in turn, to photometry has been the large amount of digital computer time required to do a point-by-point integration on a number of photographs. We shall examine in this memorandum an electrical analog which yields a slope-to-height conversion on a parallel rather than a serial basis, over the whole photograph, obviating the need for a sequential integration.

2.0 THE PRESENT, SERIAL APPROACH

Let us examine briefly the photometric analysis presently done by digital means. A photographic negative of a lunar scene, in which lunar scene brightness varies with local surface slope according to the photometric function, is scanned by a microdensitometer. An analog scan of photographic density vs. distance X is thus produced, and the densities are digitized for computer analysis. The computer then converts the density data to lunar surface slope values, using a density-to-brightness function (the film D - log E curve) and a brightness-to-slope function (the lunar photometric function). The computer then has available the function $\frac{dZ}{dX}$, where Z is local lunar surface elevation, as a function of X*.

In order to obtain the elevation profile Z(X), the computer performs a point-by-point integration. Letting Z_0 be an arbitrary elevation datum at point X_0, the height Z_1 at X_1 is obtained from:

$$\Delta Z_1 = Z_1 - Z_0 = \left.\frac{dZ}{dX}\right|_{X_1} (X_1 - X_0)$$

In general, for uniformly spaced points,

$$\Delta Z_i = Z_i - Z_{i-1} = \left.\frac{dZ}{dX}\right|_{X_i} \Delta X \tag{1}$$

$$Z_i = \sum_1^i \Delta Z_i + Z_0 \tag{2}$$

and

$$Z_i = \sum_1^i \left.\frac{dZ}{dX}\right|_{X_i} \Delta X + Z_0 \tag{3}$$

Thus to obtain the elevation at point X_i, the computer must perform i sequential integrations, the result of each integration being the required arbitrary constant for the subsequent one. This is a time-consuming and expensive procedure, particularly since using a coarse ΔX interval in the interest of speed introduces a buildup of error in Z.

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*In this memorandum we shall use the variables X and Y to represent orthogonal distances both on the lunar surface and in the photograph. Any scale factor will be ignored.

3.0 THE ELECTRICAL ANALOG, PARALLEL APPROACH

Let us instead shine a light through a positive transparent copy of the photograph, and assume that the light intensity passing through it is proportional to the slope of the lunar scene*. Now we obtain a strip of a special light-sensitive material**, whose electrical resistance varies proportionate to the light intensity striking it. We pass a constant current I through the strip, align it parallel to the X axis behind the film, and maintain point X_0 on the strip at potential V_0.

Now we obtain another strip of material, insensitive to light, having the same resistance our sensitive strip would have if it were illuminated by light through a photograph of a flat Moon (see Figure 1), and pass a current I through it also. Let us calculate the voltage difference between two corresponding points X_i of the two resistive strips:

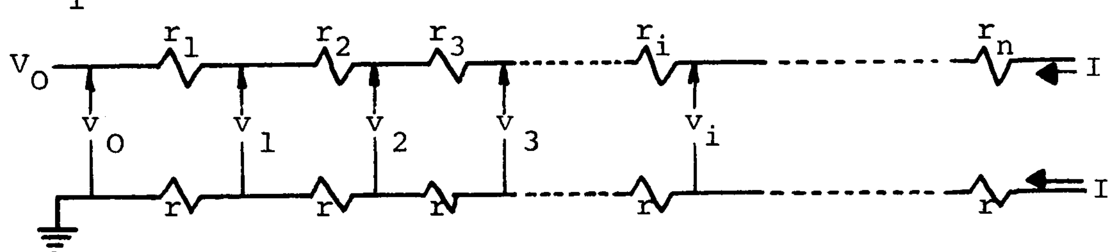

FIGURE 1

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*This is an important assumption; we shall examine it more closely in Section 4.0 of this memorandum.

**More on this later.

$$v_o = V_0$$

$$v_i = \sum_1^i Ir_i - irI + V_0 \quad (4)$$

But $r_i = r \pm \Delta r_i$

Where $\Delta r_i = 0$ for a flat local slope.

Thus $$v_i = \sum_1^i \pm I\Delta r_i + V_0 \quad (5)$$

Equations (3) and (5) are therefore directly analogous, except that the algebraic sign of Δr_i has been written out for emphasis; clearly the values of $\left.\frac{dZ}{dX}\right|_{X_i}$ of equation (3) can be positive or negative.

The reason that this analog is useful is really quite simple to understand. The light intensity passing through each interval ΔX_i of the film is proportional to $\left.\frac{dZ}{dX}\right|_{X_i}$ and thus proportional to $\left.\frac{dZ}{dX}\right|_{X_i} \Delta X$, or ΔZ_i. Thus the light passing through the film may be thought of as representing a whole row of values of ΔZ_i, each independent. By forcing each ΔV_i to be connected to its neighbors, just as the ΔZ_i height differences are on the moon, we remove the artificial independence introduced by the photographic process. The constants of integration are thus, in a sense, self-determined.

There remains, of course, the problem of a readout system. Although the voltages v_i are available simultaneously, we might still prefer to plot them sequentially, for example on a strip chart recorder connected to the resistive strips by sliding contacts.* Another possible readout would be a reconversion to brightness, producing a piece of film whose transmission varied with surface elevation instead of slope (let us note in passing that a potential-to-density conversion is just the process used in the Xerox copier). The reconversion to brightness might also be accomplished by an array of light-emitting diodes, connected to the voltages v_i. Finally, let us point out that we are more likely to find a material whose conductivity increases with incident light than one whose resistance increases; we shall illuminate such a material through a transparent negative rather than a positive of the original scene.

4.0 CALIBRATION

We have deferred the problem of the nonlinearity of the photometric function, by assuming that the light intensity passing through the lunar photographs was proportional to the surface slope. In general this is not true; the photometric function typically looks like the curve in quadrant I of Fiqure 2, where α is the surface slope.

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* Abrasion of the strip can be avoided by using an alternating current I cos wt and capacitive coupling to the recorder probe.

In an attempt to linearize the problem, let us seek a particular shape of film characteristic which will compensate for the flattening of the photometric curve as α increases in the negative direction. We require a resistance which varies linearly with slope α, as shown in quadrant IV; the reciprocal of this required curve, conductivity σ as a function of slope, is also shown. If the light which passes through the negative yields a conductivity proportional to the transmission T of the negative (quadrant III), then the T versus ϕ curve which is necessary to achieve the required linear r vs. α curve is shown in quadrant II.

More specifically, we select a value of α, and find from it the corresponding values of r and σ from quadrant IV. Using σ from quadrant IV we find the required film transmission T from quadrant III. The lunar brightness which is available to expose this film is proportional to the value of ϕ (quadrant I) which corresponds to the given α. Thus the necessary film curve T vs. ϕ (T vs. exposure) appears in quadrant II.

This film curve is plotted in more familiar terms as a D-log E curve in Figure 3, along with a curve of the toe portion of one familiar film, the Type 3400 black-and-white film used on some Apollo missions (the positioning of the curves along the exposure axis is arbitrary). Clearly the general shapes of the curves are similar, although considerable experimentation would be necessary to find a film/processing combination which would match closely.

5.0 FUNCTIONS OF TWO VARIABLES, AND OTHER APPLICATIONS

The lunar problem results in a two-dimensional array of brightnesses which are functions of a single variable, namely slope in the phase plane--the analog is a group of parallel photoconductive strips, each independent. In general we shall be interested in two-dimensional arrays which are functions of two variables; an example in the above terms would be a Moon illuminated by two Suns in orthogonal planes.

One is tempted to suggest a two-dimensional photoresistive lattice for the analog to this latter case, but a little thought (or a great deal of algebra) makes it clear that such an array would introduce a coupling between the voltage drops in the two directions. Clearly the integrator that we seek for this case is one that can integrate the total differential

$$dZ = \frac{\delta Z}{\delta X} dX + \frac{\delta Z}{\delta Y} dY \tag{6}$$

In discrete terms,

$$\Delta Z_{i+1,j+1;i,j} = \left.\frac{\delta Z}{\delta X}\right|_{i,j} \Delta X_{i+1,i} + \left.\frac{\delta Z}{\delta Y}\right|_{i+1,j} \Delta Y_{j+1,j} \tag{7a}$$

$$= \left.\frac{\delta Z}{\delta Y}\right|_{i,j} \Delta Y_{j+1,j} + \left.\frac{\delta Z}{\delta X}\right|_{i,j+1} \Delta X_{i+1,i} \tag{7b}$$

For the two-dimensional, two variable case, then, let us consider a two-dimensional array of silicon photo-diodes, each connected to its neighbors as shown in Figure 4. The voltage drop across each diode depends only on the intensity of the light striking it, and not on the adjacent diodes; the voltage drops in the two directions are therefore independent, and we may write

$$\Delta V_{i+1,j+1;i,j} = \Delta V_{i+1,i;j} + \Delta V_{i+1;j+1,j} \tag{8a}$$

$$= \Delta V_{i;j+1,j} + \Delta V_{i+1,i;j+1} \tag{8b}$$

Equations 7a and 7b are seen to be directly analogous to equations 8a and 8b.

We have left out one thing: it is necessary that brightness information representing slopes in the X direction be presented only to the appropriate diodes in a row of the array, and Y-direction information to appropriate diodes in a column of the array. Moreover, the columns and rows must be illuminated simultaneously in order for the ΔV's to add up properly. One way to do this is by filtration, exposing two negatives, one through a red filter for slopes in the Z-X plane and one through a blue filter for slopes in the Z-Y plane. The images would then be combined in a beam splitter, and additional red and blue filters used to make the row diodes sensitive only to red and the column diodes sensitive only to blue (see Figure 5).

As pointed out earlier in our discussion of the one-dimensional case, a reconversion to brightness would be possible with an array of light-emitting diodes. Note that the input potentials to these diodes should all be referenced to a common ground, since the desired output is the single variable, brightness, corresponding to the single variable, elevation.

One application of this type of integrating device would be to a two dimensional analog computer, in which the input (and perhaps output) data are presented in terms of light intensity rather than voltage. Another of many necessary blocks of such a system would be an input device, or function generator. For cases in which a physical model already exists, the lunar photometry technique described earlier may be useful. For example, consider a laboratory model of an airplane wing's upper surface. A powder such as cupric oxide, which exhibits a degenerate photometric function similar to that of the Moon, could be dusted on the wing surface, and photographs taken of the result. Applying the resulting photos as inputs to the above integrator could result in a potential or brightness map representing the wing's surface. The potentials or brightnesses could then be manipulated in the computer to represent deflections, distortions, etc.

6.0 CONCLUSIONS

A very preliminary look at analog techniques has led to a suggested way to simplify the reduction of lunar photometric data to lunar surface elevation data. The approach has been generalized to functions of two variables, leading to a possible integrator for a presently non-existent two-dimensional analog computer. Given the present state of the art of handling large amounts of two-dimensional data, this could well be worth pursuing. Considerable hardware development would be required.

7.0 ACKNOWLEDGEMENTS

I would like to thank D. B. James for raising some initial questions, and S. Y. Lee and G. C. Reis for some helpful discussions. I remain responsible for the results.

HW Radin

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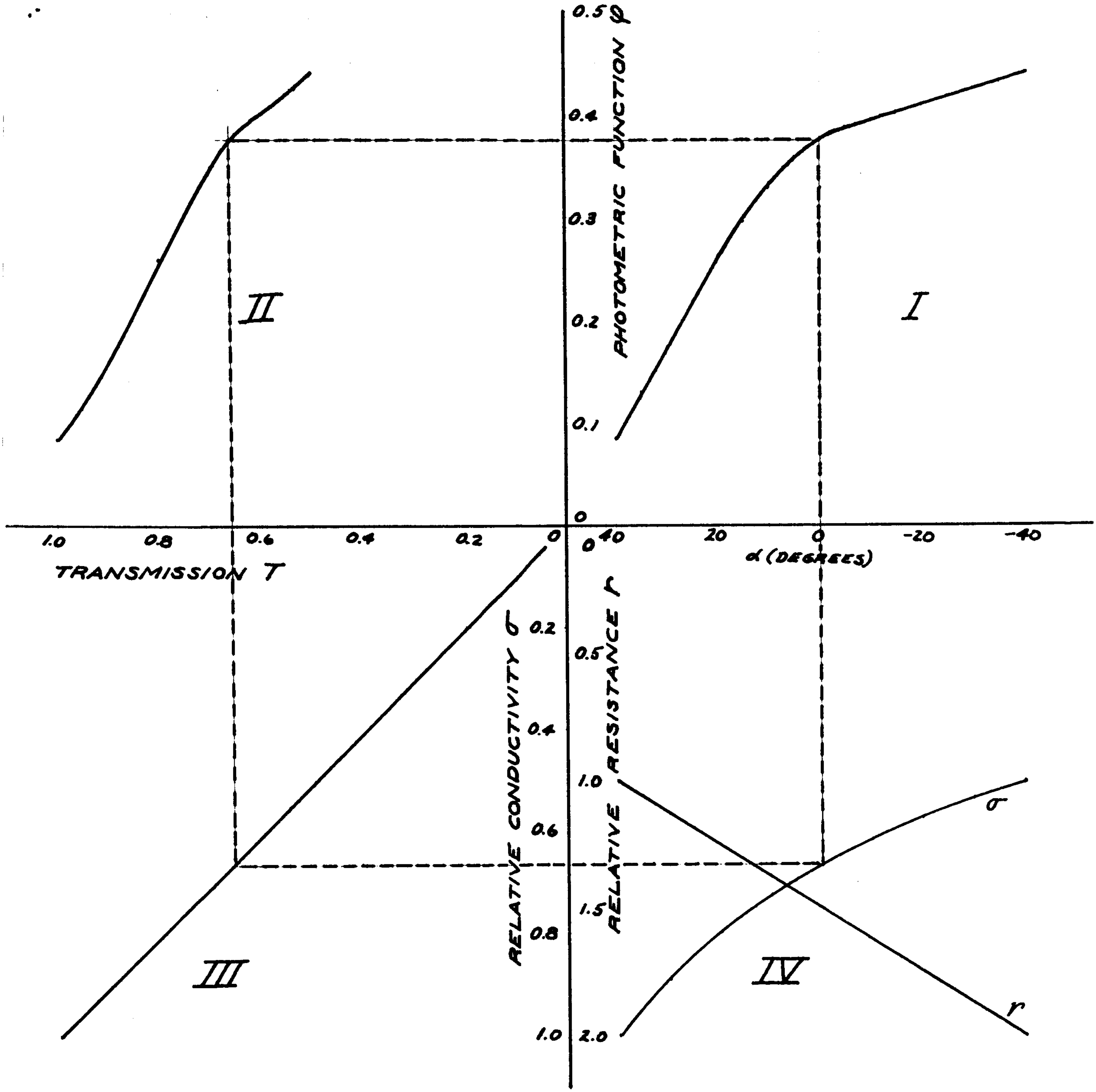


GRAPHICAL DERIVATION OF REQUIRED FILM CHARACTERISTIC

FIGURE 2

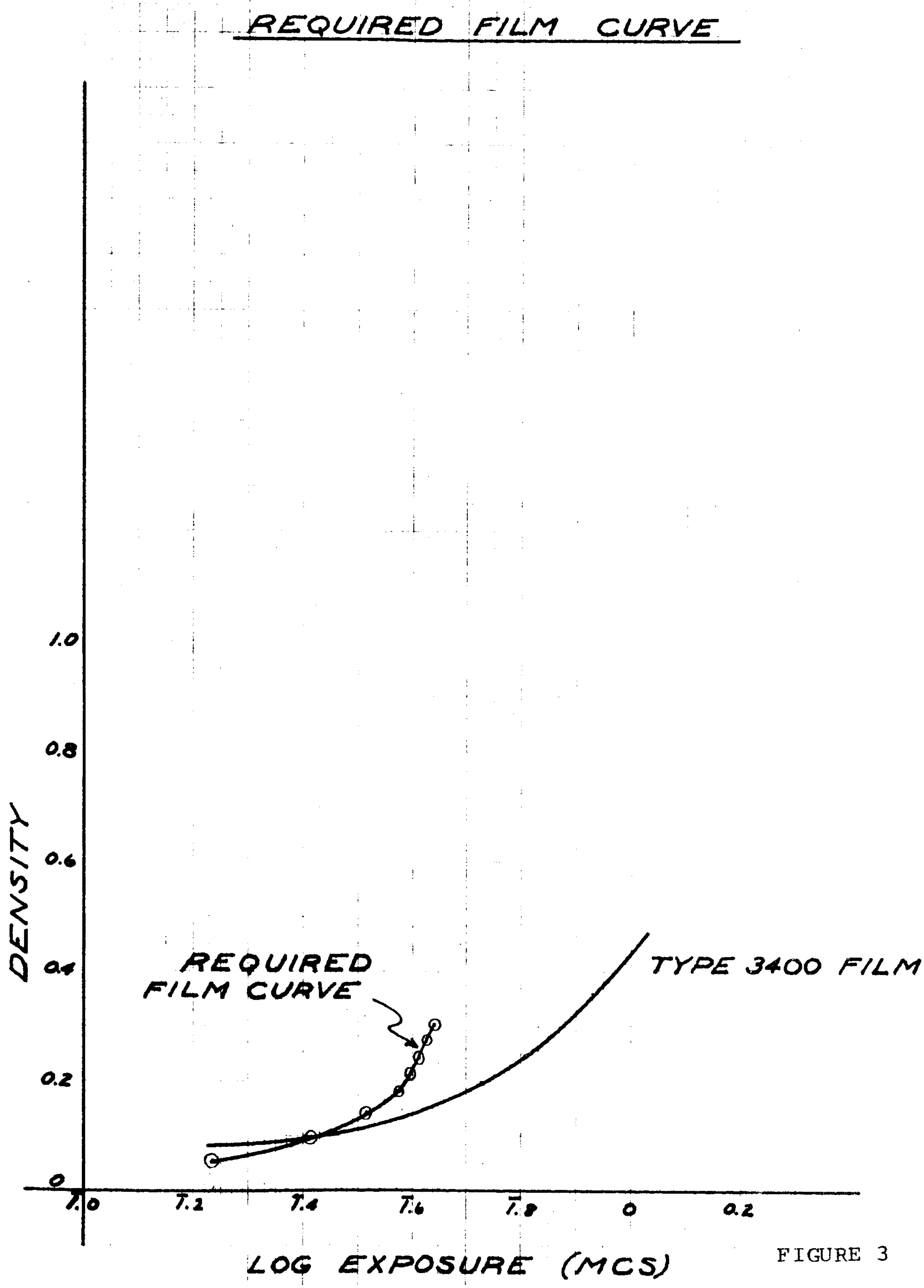
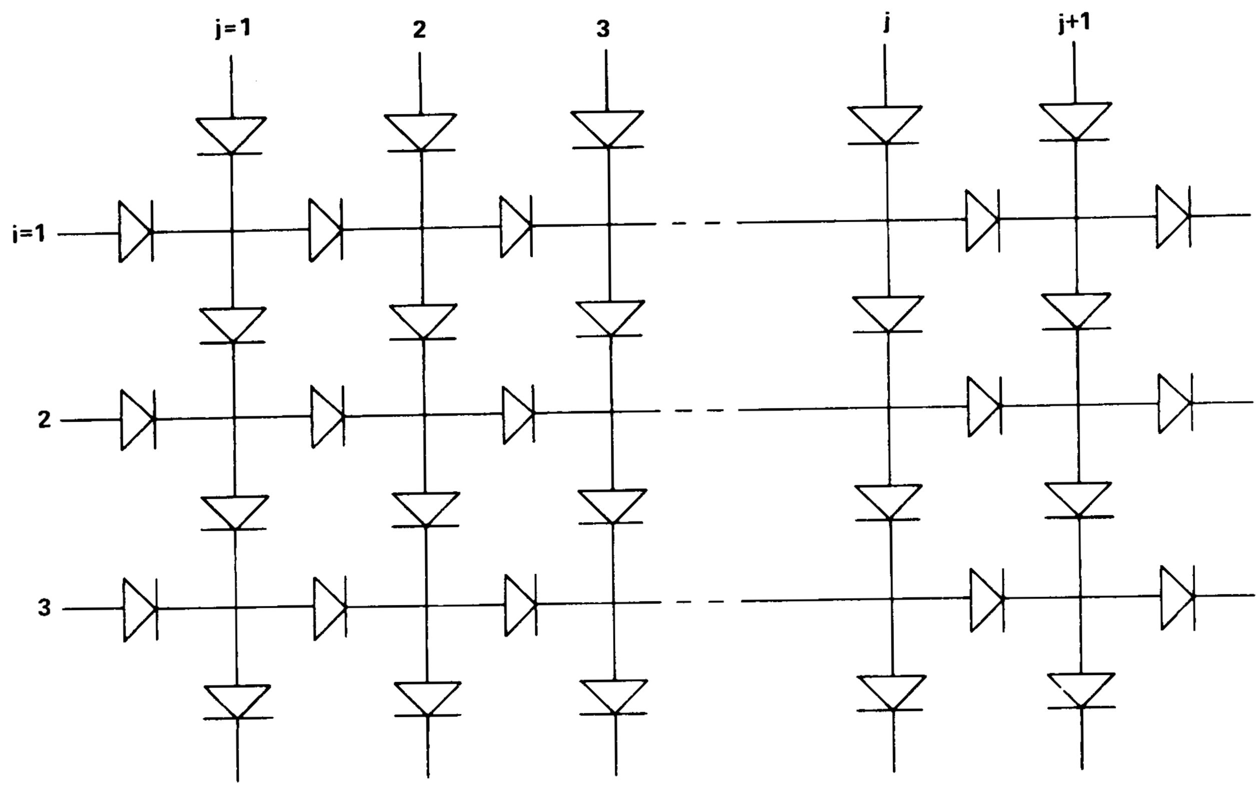

REQUIRED FILM CURVE
DENSITY
1.0
0.8
0.6
0.4
0.2
0
REQUIRED FILM CURVE
TYPE 3400 FILM
$\bar{1}.0$
$\bar{1}.2$
$\bar{1}.4$
$\bar{1}.6$
$\bar{1}.8$
0
0.2
LOG EXPOSURE (MCS)

FIGURE 3



TWO – DIMENSIONAL DIODE POTENTIAL ANALOG

FIGURE 4

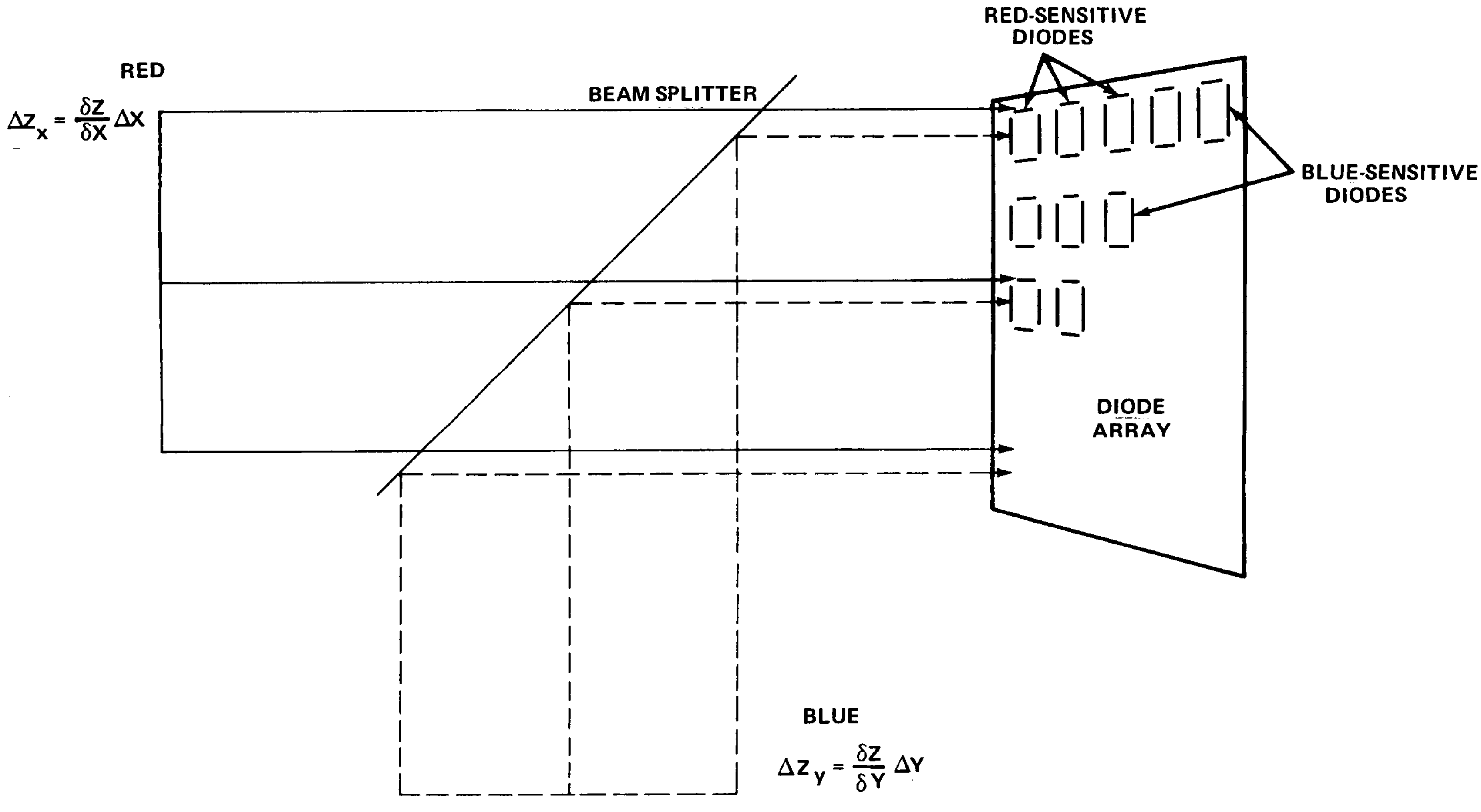


FIGURE 5- DIODE ANALOG SHOWING COMBINATION OF X AND Y DIRECTION ELEVATION DIFFERENCES